Work and the Farm

By Michael S. Fry, Palmer High School

Materials
- classroom with overhead or chalkboard for presentation of the problems.
- Optional materials: ramp, lever, pulley, gear, screw, wedge, a car "stuck" in the mud with rope and a pole nearby, a bicycle, pulleys set up as per the assignments.

Objectives
Give physics students a summary or review of the concepts of work, force, torque, velocity, equilibrium as applied to agriculture.

Suggested grade levels
11-12

Alaska Content Standards
A.6, A.8 (standards B.1 and B.4 could be applicable if the lesson was used to investigate how the objects worked).

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Background

When you think of the farm, the idea of work often comes to the forefront. So if there's so much work on a farm, is there a way of making that work easier? Sure, just apply sound principles of physics. Physics states that work is the dot product of force times displacement, and that work changes an object's energy. So in other words, if I need to change an object's energy, I do it through applying work. When would I need to change the energy of an object? Whenever I wish to change the object's position in a force field or whenever I want to speed it up. So how do I do it the "easy" way. Well, an energy change takes a certain amount of work no matter what (that can't be changed), but I can change HOW I do the work. In other words, I can change either the distance over which I apply my force, or the amount of force I apply. Well, for us weaklings, I'd rather apply less force through a greater distance (endurance instead of strength). Are there things that help me use less force through a greater distance? Yes, we call them machines. A machine, no matter how complex, is made up of one of the following four simple machines: 1) the pulley 2) the ramp, 3) the lever, 4) and the gear (the fifth and sixth machines, a screw and a wedge, won't be explored in the following situations). Each of the machines uses principles of physics to tradeoff one quantity for another.

Let's look at each one. The pulley merely changes the direction of the force vector you need to apply in any particular application. For example, you wish to raise a load of hay to a floor above you. You could simply lift the load using your arms (which aren't as strong as your back or legs) to get it above your head and then heave the load upward or you could attach a pulley to the floor above you, hook your load to a rope and loop the rope through the pulley, and simply pull down on the load with your weight—you simply need to hold onto the rope. But what if your weight is less than the load? You won't be able to lift the load by hanging onto the rope. You could attach another pulley to your floor, loop the rope through that pulley, and now pull up with your back and legs (of course you'll still need to hold on with your hands, but at least you won't have to pull and heave with your arms). But what if you don't have that strength (force)? How about a ramp?

The ramp doesn't change the direction of the force vector like a pulley, it actually uses the principle of doing work by increasing the distance over which the force is applied. Let's suppose you have a ramp that is inclined 5 degrees. How much force is needed to lift a 100 kg bale of hay to the floor above you (5 meters)? Well, the change in potential energy of the hay is given by U = mgh with m = 50 kg, g = 9.8 m/s2, and h = 5 m.
The result is U = (5kg)(9.8m/s2)(5m) = 2450J. How much work is needed? The same. Now, if the ramp is inclined 5 degrees, and is frictionless, over what distance will my force be applied? Well, the distance is the hypotenuse of the triangle, with the height being 5 meters. The sine of the angle of 5 degrees gives height over the hypotenuse. Therefore, the hypotenuse is 5 meters divided by the sine of 5 degrees; that all works out to be 57.4 meters. So how much force do I need to slide the bale up the ramp? Work divided by distance = force or in this case, 2450J/57.4m = 43 newtons. That's a far cry from the 490 newtons required to lift the bale. Of course, there's no free lunch; by using the ramp, the force has to be

applied over a longer distance—you'll have to walk up the ramp. But if you can't lift the hay, you're in business.

But what if the ramp has friction on it (real world problems do). Then what about rolling the hay up? Now we take advantage of one other work trade-off – the idea of torque. Torque is turning force, and is measured by the cross product of the moment arm and the force vector. The mathematical equation looks like torque = R X F or RF times the sine of the angle between the force and radius vector. If I want to turn an axle, I would attach a wrench and apply a force as far away from the center of the axle and at a 90 degree angle between the wrench and my arm. This would maximize my torque. For example if my wrench is .5 meters in length, and I can apply a force of 200 newtons with my hand, as long as I apply my hand 90 degrees to the wrench, the torque I can develop is (200 n)(.5 m) or 100 newton-meters. This principle of torque is needed to determine the usefulness of the lever.

Let's suppose that I have a lever of length 5 meters, and I want to lift a diesel engine of 1000 newtons on that lever. If I can place the fulcrum of the lever 1 meter from the load, and I can apply my force 90 degrees to the lever at the other end of the lever, the laws of equilibrium state that the load won't accelerate if my torque equals the torque of the load. The torque of the load is the lever arm times the force of the load (its weight) or in this case (1m)(1000n) = 1000n m. If that equals my torque which is my lever arm times my force, then I can calculate the needed force to be the torque divided by my lever arm of 4 meters. Thus my force is (1000nm)/(4m) = 250 newtons, and thus I can lift a diesel engine which weighs 1000 newtons with only a 250 newton force. But as always, there's no free lunch; I have to apply that force over a greater distance. Go back to the idea of work. If the engine is raised 1 meter, it gains 1000 joules of energy, and I must apply my 250 newtons through a distance of 4 meters, but if by doing so, the required force is reduced by a factor of 4, then I've succeeded in my task.

Not only can I trade-off distance for force, I can also trade-off velocity for torque, and that leads us to the idea of the gear. A gear simply increases or decreases the tangential velocity (linear speed) of an object by increasing or decreasing the force applied to the gear. If a rope goes around a gear of radius 5 cm and that gear is bolted to a smaller gear of 2.5 cm, then the rope going around the larger gear will travel twice as fast in the same amount of time as the smaller gear. The principle is that the angular velocity of both gears is the same (they both rotate at the same angular rate—they go around the same number of degrees per second), and tangential velocity is angular velocity times the radius vector. Thus if the angular velocity is 100 rpm, then the velocity of the rope around the larger gear is (100 rev/min)(1 min/60 sec)(2pi radian/1 rev)(.05 m) = 10.4 m/s; whereas, the velocity around the smaller gear is half that, or 5.2 m/s. This change in velocity doesn't come without a cost though. In order to turn the gear, there must be a torque involved, and torque is force times radius. Since the torque on both gears is the same, the needed force on the smaller gear is greater than on the larger gear. Or in other words, if I want a larger velocity outcome, I need a greater force input. So what if I need to stir a vat of milk at 30 rpm with a paddle with radius 1 meter, the force of the friction on the paddle is 50 newtons, and the radius of the paddle axle is .01 m. How much torque is required? And at what velocity? The torque would be (50 n)(1 m) or 50 nm which would require (50 nm)/(.01 m) = 5,000 newtons acting on the axle, and the velocity of the rope around the axle would be (30 rpm)(1/60(2pi)(.01 m) = .03 m/s. But suppose my motor that can only deliver 100 newtons of force, but at 1.5 m/s, what could I do? Could I use a gear to still turn the milk stirrer? I would need a gear mounted on the axle that would be .5 meter in radius. This would deliver the needed

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torque, and rpm—the force of the motor times the gear radius equals the needed torque (100 n)(.5 m) = 50 nm, and the velocity divided by the radius equals the rotational velocity (.5 m/s)/(.5 m)/(2pi)(60 s) = 29 RPM. The principle is the same as on a 10 speed bicycle. Your back wheel is large in radius, so for a large tangential velocity of the bicycle, the back gear of the bicycle, having a much smaller radius, means that the back gear speed is much smaller than the rim of the tire. Its torque requirement is much higher though. That torque comes from the chain looped around the front sprocket. Because the front sprocket is larger than the back gear, it must go around faster, but its torque requirement is lower. The torque is developed from the pedal's lever arm and the force of your leg pushing on the pedal. So if you want to go faster you could change the front sprocket to a larger gear, which would increase the amount of torque required, and thus the harder you would have to push on the pedal, but the greater your speed. Or you can decrease the rear gear size which will increase its force requirement, but give you greater tire speed for a given gear speed. Thus you can push with a fairly low velocity on your leg and translate that into a high velocity of your bicycle. Conversely, if you're going up a hill, and need to reduce your force requirement, you'll pay for it in slowed forward velocity because of higher gear velocity.

Now let's revisit the pulley and apply the idea of torque to it. Not only can it change the direction of the force vector, it can also be used in such a way that I decrease my force, but increase the distance over which I apply that force (sound familiar). The principle used is that of equilibrium. In order for equilibrium to apply, the net force AND the net torque acting on an object must be zero. For a pulley that means that the torque on one side of the pulley, must be the same as the torque on the other side. But if the radius on one side of the pulley is that same as the other side of pulley, then the forces acting on the pulley from the rope must be equal and they must then oppose the force on the axle of the pulley. Thus I can get two forces acting in one direction (assuming the pulley is anchored to the floor or ceiling) for the one force I apply (that sound's like something for nothing), but the trade-off is that I have to pull on the rope twice the distance. By stringing my pulleys in series, I can further increase the distance the rope I'm pulling on must go through in order for the pulleys to deliver their work.

Now apply the above principles of work, torque and equilibrium to solve the following problems.
See separate sheets.

SOLUTIONS

1. a. Frope = 2450 n, Fanchor = 12250 n 4Frope = 9800 n
   b. Frope = 1960 n, Fanchor = 7840 n 5Frope = 9800 n
   c. Frope = 1400 n, Fanchor = 11200 n 7Frope = 9800 n
   d. Frope = 1633 n, Fanchor = 8165 n 6Frope = 9800 n
   set up D is the only one that will work without adding mass to your body.

2. For the set up with a bar through the center of the bale:
   Mgsin(5 degrees) = RF, therefore F = MGsin(5 deg.) = 85 n
   For the set up with you pushing on the top of the bale:
   Mgsin(5 degrees) = 2RF, therefore F = MGsin(5 deg.)/2 = 43 n

3. (Fleaning post)Cos(75 deg.) = Ftruck and Fup on trunk = Fleaning postSin(75 deg.) therefore Fup on trunk = FtruckTan(75 deg.) = (1000n)Tan(75 deg.) =
3730 n

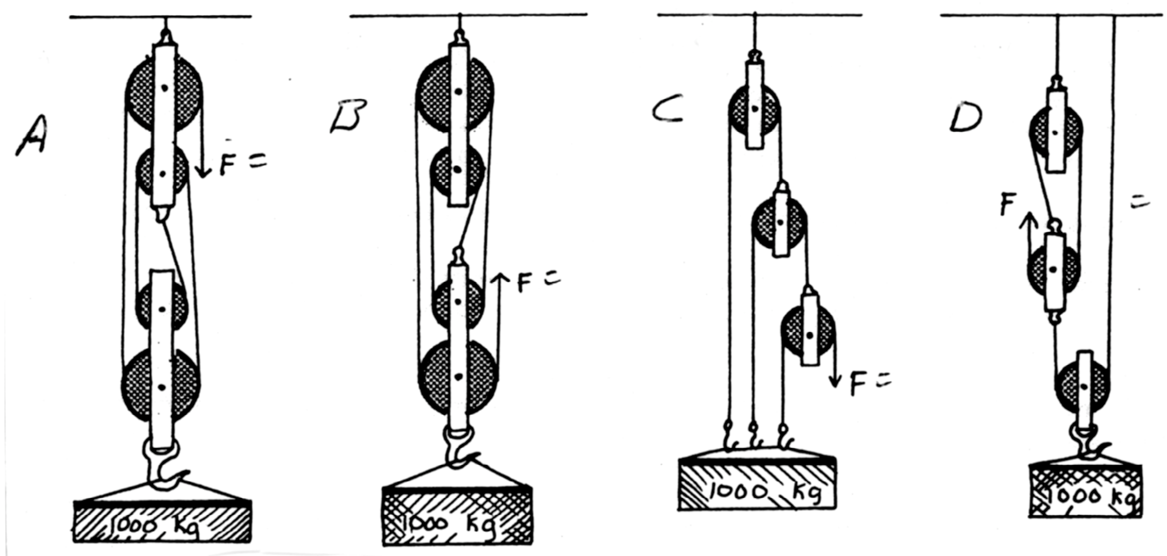
4. 2Tension in ropeSin(5 deg.) = force by individual
therefore T = F/2Sin (5 deg.)

   For standing on rope F = 1000 n, therefore T = 5740 n
   For pushing horizontally F = 500 n, therefore T = 2700 n
   For pushing up vertically F = 2000 n, therefore T = 11,500 n

Push up with your back

Name____

Work and the Farm Problem Set

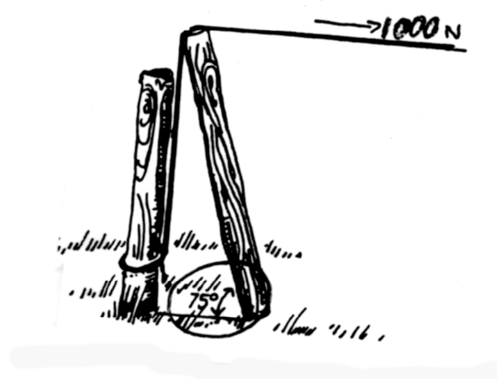


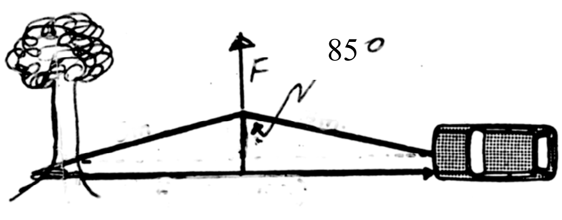
1. You need to lift a diesel engine with a mass of 1000 kg out of your truck. Use the following pulley arrangements to determine the force required on the rope and the anchor and which one will best meet your limitations (assume your mass to be 100kg and you can pull up with a force of 1700 n).

2. The friction on the ramp used at the beginning of the lesson is much more than the force you can deliver, but what if you rolled the bale up the ramp? Suppose the bale has a radius of 1 meter, what would the force be required to roll it up the ramp (assume no rolling friction) if you inserted a bar in the center of the bale and pulled the bale up the ramp? What would be the force required if you rolled the bale up by applying the force to the top of the bale? (5 degree angle, 100 kg bale)

Work and the Farm Problem Set

3. There are a lot of stumps on the farm that need pulling. What if you used the arrangement below to pull the stump. How much force would be delivered on the stump if your truck can deliver 1000 newtons of force on the rope?

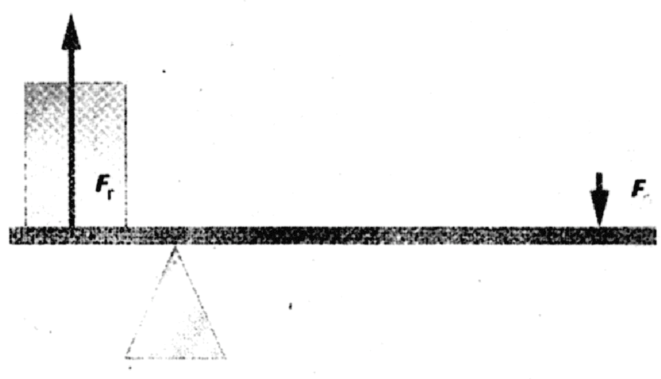




4. Sometimes you get stuck in the mud with your tractor, but you always bring along a rope, and beside the muddy area is a row of trees. How could you increase your body's pulling or pushing force by using the rope and a nearby tree? Would you want the rope taunt or slack? Where would you apply your force (near the end of the rope or the middle)? What if you couldn't get much traction, could you apply your force some other way that pushing or pulling horizontally? Would you develop more force by standing on the rope or pulling up (Is the force of pushing down on something limited by your body's mass? What is the limiting force of your body pulling up on something?) Assume friction only allows you to push horizontally with a force of 500 newtons, but you can push with 2000 newtons upward, and you weigh 1000 newtons. Also assume the angle formed between the rope and the line between the tractor and the tree is 85 degrees once you apply a force to the rope. Calculate the force each method will produce on the tractor. Which method will produce the maximum force to get your tractor out of the mud?

Simple Machines

3)



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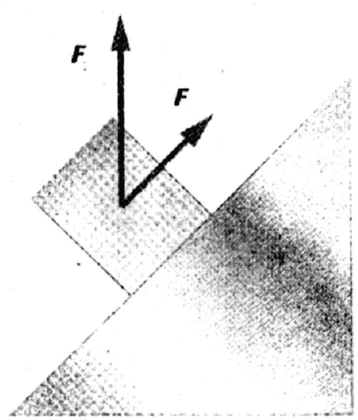

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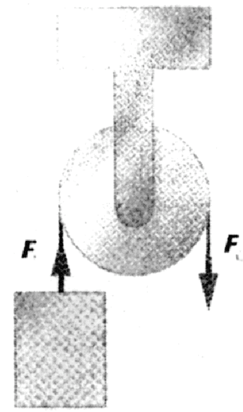

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